VU/PG Adm./15/19

VIDYASAGAR UNIVERSITY

MIDNAPORE

COMMON ENTRANCE TEST FOR PG ADMISSION, 2019

Question Booklet No. 1514430 — Full Marks : 200

Subject: APPLIED MATHEMATICS — **Question Booklet Series: A**

Subject Code No.: 15

Answer all the questions. Each question has the same weightage.

Read the following instructions carefully before you start answering.

INSTRUCTIONS

1. The question Booklet is printed in four Series e.g. (A), (B), (C) and (D). The candidate has to indicate the Series of the question booklet in the space provided in the OMR Answer Sheet . For example, if the candidate gets Series (A) booklet, he / she has to indicate on the front side of the OMR Answer Sheet with Black ink ball point pen only as indicated below:

   ● Ⓑ Ⓒ Ⓓ

2. There are 50 questions inside this question booklet. Immediately after you have been instructed to open this question booklet, ensure that any page / question is not missing / not printed / torn /repeated. In case you find any defect anywhere in the question booklet, immediately get it replaced by the Invigilator.
3. Each question carries 4 marks. 1(one) mark will be deducted for each wrong answer(negative marking).
4. Write your Form No and put signature in the space provided.
5. Before answering, write down the necessary information on the OMR Answer Sheet as per your Application Form and Admit Card in the specific space provided.
6. With each question you will find 4 possible answers marked by the letters A, B, C & D. Read each question carefully and find out which answer, according to you, is correct / most appropriate / best. Indicate your answer by darkening the appropriate circle completely in the OMR Answer Sheet corresponding to the question. For marking answers, use black ink ball pen only. If 'B' is the correct answer in a case, mark as below:

   Ⓐ ● Ⓒ Ⓓ

7. Do not fold or make any stray marks on the OMR Answer Sheet.
8. You can use the blank space of the last page for rough work. Do not tear it off from the Question Booklet.
9. After the examination has been over, you must submit OMR Answer Sheet to the Invigilator.
10. OMR Answer Sheet is designed for computer evaluation. If you do not follow the instructions given above and shown in the OMR Answer Sheet, it may make evaluation by computer difficult. Any resultant loss to the candidate on the above account shall be of the candidate only.
11. No candidate shall be allowed to use Mobile phone. Log tables or Calculator of any description in the examination hall / room.

1. The second central moment of the Binomial distribution $B(1, \frac{1}{2})$ is

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{8}$

2. If $\sum x = 30$, $\sum y=42$, $\sum xy=199$, $\sum x^2 = 184$, $\sum y^2 = 318$ and $n = 6$, then the regression coefficient b_{xy} is

(A) -0.36 (B) -0.46 (C) 0.26 (D) 0.38

3. A regression model is used to express a variable Y as a function of another variable X. This implies that

(A) there is a causal relationship between X and Y.
(B) a value of X may be used to estimate a value of Y
(C) a value of X exactly determine a value of Y
(D) there is no causal relationship between X and Y

4. Let X be a non-negative integer valued random variable with E(*X*) = 1.Then the value of $\sum_{i=1}^{\infty} P[X \geq i]$ is

(A) 0 (B) 2 (C) 1 (D) none of these

5. Let V be the volume of a region bounded by a smooth closed surface S. Let $\vec{r}$ denotes the position vector and $\hat{n}$ denotes the outward unit normal to S. Then the integral $\int_S \vec{r}.\hat{n}\, dS$ equals to

(A) $2V$ (B) $5V$ (C) $3V$ (D) $4V$

6. The area of a parallelogram having diagonals $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$ is

(A) $\frac{5\sqrt{3}}{2}$ sq. unit (B) $5\sqrt{3}$ sq. unit (C) $10\sqrt{3}$ sq. unit (D) $\frac{5\sqrt{3}}{7}$ sq. unit

7. For a second order tensor field *T*, div (curl *T*) is equal to

(A) div (div *T*) (B) curl (div T^T) (C) curl (div *T*) (D) div (div T^T)

8. The binary representation of $(37.65625)_{10}$ is

(A) $(100101.10101)_2$ (B) $(100101.01010)_2$
(C) $(110010.10101)_2$ (D) $(100101.10001)_2$

9. Magnetic Tape is a

(A) Random access medium (B) a parallel access medium
(C) universal access medium (D) serial access medium

10. The critical point of the system $\frac{dx}{dt} = -4x - y$ and $\frac{dy}{dt} = x - 2y$ is an

(A) asymptotically stable node (B) unstable node
(C) asymptotically stable spiral (D) unstable spiral

11. The logistic model is

(A) $\frac{dx}{dt} = -rx$ (B) $\frac{dx}{dt} = rx - ax^2$ (C) $\frac{dx}{dt} = re^{-xt}x$ (D) none of these

12. A square matrix A is orthogonally diagonalizable

(A) iff A is symmetric (B) if A is symmetric
(C) iff A is skew symmetric (D) if A is skew symmetric

13. Let V be the vector space of all real polynomials $p(x)$. Let D and T be the linear mappings of V to V defined by $D(p(x)) = \frac{d}{dx}p(x),\ p(x) \in V$ and $T(p(x)) = \int_0^x p(t)\,dt,\ p(x) \in V$. The null space of $T \circ D$ is

(A) the subspace of all constant polynomials
(B) the subspace of all polynomials
(C) the subspace of all zero polynomials
(D) none of these

14. Let T be the linear operator on $\mathbb{R}^2$ which is represented in the standard ordered basis by the matrix $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. The characteristic values of A are
(A) i and $-i$
(B) 1 and -1
(C) 0 and 0
(D) none of these

15. The solution of the differential equation $\tan x\, dy - \tan y\, dx = 0$, given that $y = \frac{\pi}{2}$ when $x = \frac{\pi}{4}$, is
(A) $|\sin y| = \sqrt{2}|\sin x|$
(B) $\sin y = \sqrt{2}|\sin x|$
(C) $|\sin y| = \frac{1}{\sqrt{2}}|\sin x|$
(D) $\sin y = \frac{1}{\sqrt{2}}|\sin x|$

16. At each point (r, θ) the trigonometrical tangent of the angle between the radius vector and the tangent is equal to $\frac{1}{3}$ of the trigonometrical tangent of the vectorial angle. The differential equation of the curves determined by the above condition is
(A) $r\frac{d\theta}{dr} = \frac{1}{3}\tan\theta$
(B) $\frac{d\theta}{dr} = r\tan\theta$
(C) $\frac{dr}{d\theta} = 3r\tan\theta$
(D) $\frac{dr}{d\theta} = \frac{1}{3}r\tan\theta$

17. Let $L: \mathbb{R}^4 \to P_3$ be a linear transformation defined as
$L(x_1, x_2, x_3, x_4) = x_1 + (x_2 - x_3)t + (x_1 - x_3)t^3$. Dimension of Kernal of L is
(A) one
(B) two
(C) three
(D) four

18. With standard notations, the partial differential equation of the family of spheres of radius 3 with centres on the plane $y = x$ is
(A) $(x-y)^2(p^2+q^2+1) = 9(p-q)^2$
(B) $(x+y)^2(p^2+q^2-1) = 9(p-q)^2$
(C) $(x-y)^2(p^2+q^2+1) = 9(p+q)^2$
(D) $(x+y)^2(p^2+q^2+1) = 9(p-q)^2$

19. Consider a system of forces $\boldsymbol{F_1}, \boldsymbol{F_2}, \boldsymbol{F_3}, \ldots$ acting on a rigid body at the respective points $P_1, P_2, P_3, \ldots$ defined by the respective position vectors $\boldsymbol{r_1}, \boldsymbol{r_2}, \boldsymbol{r_3}, \ldots$ etc. The necessary and sufficient conditions for the static equilibrium of a rigid body are that the resultant force-couple system becomes zero such that
(A) $\sum \boldsymbol{F_i} = 0$ and $\sum \boldsymbol{r_i} \times \boldsymbol{F_i} = 0$
(B) $\boldsymbol{F_i} = 0$ and $\sum \boldsymbol{r_i} \times \boldsymbol{F_i} = 0$
(C) $\sum \boldsymbol{F_i} = 0$ and $\boldsymbol{r_i} \times \boldsymbol{F_i} = 0$
(D) None of these

20. The center of pressure of a square lamina immersed in a fluid with one vertex in the surface and the diagonal vertical divides in the ratio:
(A) 7:3
(B) 7:5
(C) 3:5
(D) 7:7

21. If u and v ($\neq 0$) are complex conjugate of each other, then
(A) $\frac{u}{v}$ must be a real number
(B) $u^2 + v^2$ must be a real number
(C) $u^2 - v^2$ must be a real number
(D) None of these.

22. The value of $\lim_{z \to 0}\left(\frac{\sin z}{z}\right)^{\frac{1}{z^2}}$ is
(A) Does not exist
(B) $e^{\frac{1}{6}}$
(C) $e^{-\frac{1}{2}}$
(D) $e^{-\frac{1}{6}}$

23. The exact value of $\int_{-1}^{1} |2x|dx$ can be numerically computed with spacing $h = 1$ using
(A) Trapezoidal rule
(B) Simpson's $\frac{1}{3}$ rule
(C) Both (A) & (B)
(D) None of these.

24. Maximum curvature of the parabola $y = ax^2$ $(a \neq 0)$ exist at the point
(A) (0, a) (B) (a, 0) (C) (a, a) (D) (0, 0)

25. If the equation $xy + z^3x - 2yz = 0$ defines z as a function of two independent variables x and y, and the partial derivatives exist, then the value of $\frac{\partial z}{\partial x}$ at the point (1, 1, 1) is
(A) -2 (B) 2 (C) 1 (D) -1

26. The sequence $\{x_n\}_{n=1}^{\infty}$ where $x_n = 1 + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} - 2\sqrt{n}$ is
(A) decreasing (B) increasing (C) constant (D) oscillating

27. Consider differentiable functions $f: \mathbb{R} \to \mathbb{R}$ with the property that for all $a, b \in \mathbb{R}$, we have $f(b) - f(a) = (b - a)f'(\frac{a+b}{2})$. Then which of the following statement is true?
(A) Every such f is a polynomial of degree less than or equal to 2
(B) There exist such a function f which is a polynomial of degree bigger than 2
(C) There exist such a function f which is not a polynomial
(D) Every such f satisfies the condition $f\left(\frac{a+b}{2}\right) \leq \frac{f(a)+f(b)}{2}$ for all $a, b \in \mathbb{R}$.

28. Let $f: (0, \infty) \to \mathbb{R}$ be defined by $f(x) = \frac{\sin(x^3)}{x}$. Then f is
(A) bounded and uniformly continuous (B) bounded but not uniformly continuous
(C) not bounded but uniformly continuous (D) not bounded and not uniformly continuous

29. Consider functions $f: \mathbb{R} \to \mathbb{R}$ with the property that $|f(x) - f(y)| \leq 4321\,|x - y|$ for all real numbers x, y. Then which of the following statement is true?
(A) f is always differentiable
(B) There exist at least one such f that is continuous and such that $\lim_{x\to\pm\infty} \frac{f(x)}{|x|} = \infty$
(C) There exist at least one such f that is continuous, but is non-differentiable at exactly 2018 points and satisfies $\lim_{x\to\pm\infty} \frac{f(x)}{|x|} = 2018$
(D) It is not possible to find a sequence $\{x_n\}_{n=1}^{\infty}$ of real numbers such that $\lim_{n\to\infty} x_n = \infty$ and further satisfying $\lim_{n\to\infty} \left|\frac{f(x_n)}{x_n}\right| \leq 10000$

30. Let $A = \left\{\sum_{i=1}^{\infty} \frac{a_i}{5^i} : a_i = 0, 1,\ 2, 3\ or\ 4\right\} \subset \mathbb{R}$. Then
(A) A is a finite set
(B) A is countably infinite
(C) A is uncountable but does not contain an open interval
(D) A contains an open interval

31. If $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ are arbitrary points in the plane define the metric $d(P, Q) = \max\{|x_1 - x_2|, |y_1 - y_2|\}$. Let $P = \left(2, \frac{1}{2}\right)$ and $S = [0, 1] \times [0, 1]$. Which of the following statement is true?
(A) There does not exist any point $Q \in S$ such that $d(P, Q) = \min\{d(P, X) : X \in S\}$
(B) There exists a unique point $Q \in S$ such that $d(P, Q) = \min\{d(P, X) : X \in S\}$
(C) There exist infinitely many points $Q \in S$ such that $d(P, Q) = \min\{d(P, X) : X \in S\}$
(D) None of the above.

32. Let A be the set of all continuous functions $f: [0, 1] \to [0, \infty)$ satisfying the condition: $\int_0^x f(t)dt \geq f(x)$ for all $x \in [0, 1]$. Then which of the following statement is true?

(A) A has cardinality 1 (B) A has cardinality 2 (C) A is infinite (D) A is empty

33. Consider the following four sets of maps
(i) $\{f: \mathbb{Z} \to \mathbb{Q}: f$ is bijective and increasing$\}$
(ii) $\{f: \mathbb{Z} \to \mathbb{Q}: f$ is onto and increasing$\}$
(iii) $\{f: \mathbb{Z} \to \mathbb{Q}: f$ is bijective and satisfies that $\forall n \leq 0, f(n) \geq 0\}$
(iv) $\{f: \mathbb{Z} \to \mathbb{Q}: f$ is onto and decreasing$\}$
How many of these sets are empty?

(A) 0 (B) 1 (C) 2 (D) 3

34. Let $f: [-\pi, \pi] \to \mathbb{R}$ be a continuous 2π-periodic function whose Fourier series is given by $\frac{a_0}{2} + \sum_{k=1}^{\infty}(a_k cos kt + b_k sin kt)$.
Let for each $n \in \mathbb{N}$, $f_n(t) = \frac{a_0}{2} + \sum_{k=1}^{n}(a_k cos kt + b_k sin kt)$ and let f_0 denote the constant function $\frac{a_0}{2}$.
Which of the following statement is true?

(A) $f_n \to f$ uniformly on $[-\pi, \pi]$
(B) If $\sigma_n = \frac{f_1+f_2+\cdots+f_n}{(n+1)}$, then $\sigma_n \to f$ uniformly on $[-\pi, \pi]$
(C) $\int_{-\pi}^{\pi} |f_n(x) - f(x)|^2 dx \to 0$ as $n \to \infty$
(D) None of the above.

35. Which of the following statement is true?
(A) The sequence of functions $\{f_n\}$ defined by $f_n(x) = x^n(1-x)$, is not uniformly convergent on the interval $[0, 1]$.
(B) The sequence of functions $\{f_n\}$ defined by $f_n(x) = n \log(1 + \frac{x^2}{n})$, is uniformly convergent on $\mathbb{R}$.
(C) The series $\sum_{n=1}^{\infty} 2^n \sin(\frac{1}{3^n x})$ is uniformly convergent on the interval $]1, \infty[$.
(D) None of the above.

36.
The basis matrix of the system $\begin{pmatrix} 3 & 2 & 5 \\ 1 & 4 & 10 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ is

(A) $\begin{pmatrix} 3 & 5 \\ 1 & 10 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (C) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (D) $\begin{pmatrix} 2 & 5 \\ 4 & 10 \end{pmatrix}$

37. The value of the following 2×2 game

$$\begin{array}{cc} & \text{Player A} \\ \text{Player B} & \begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix} \end{array}$$ is

(A) $\frac{5}{6}$ (B) $\frac{1}{6}$ (C) $\frac{7}{3}$ (D) $\frac{3}{7}$

38. The solution of the following transportation problem

	D_1	D_2	D_3	D_4	a_i
O_1	4	2	7	-1	27
O_2	3	0	2	4	33
O_3	5	3	4	5	23
O_4	3	5	4	-2	17
b_j	31	24	25	20	

is $x_{11} = 24$, $x_{14} = 3$, $x_{21} = 7$, $x_{22} = 24$, $x_{23} = 2$, $x_{33} = 23$ and $x_{44} = 17$. The nature of this solution is
(A) non-degenerate and unique (B) degenerate and unique
(C) degenerate and not unique (D) non-degenerate and not unique

39. The equation of the common tangent between the circle $x^2 + y^2 = 4x$ and the parabola $y^2 = 4x$ is
(A) $y = x + c$ (B) $y = 0$ (C) $x = 0$ (D) $y = x$

40. The angle at which the axes are to be rotated so that the equation $x\sqrt{3} + y + 6 = 0$ may be reduced to the form $x = 3$ is
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

41. The projection of the line segment, joining the points $(3, 3, 5)$ and $(5, 4, 3)$, on the straight line, joining the points $(2, -1, 4)$ and $(0, 1, 5)$, is
(A) $\frac{3}{4}$ (B) $-\frac{3}{4}$ (C) $-\frac{4}{3}$ (D) $\frac{4}{3}$

42. The equations of the generators of the hyperboloid $x^2 - y^2 = 2z$ passing through the point $(5, 3, 8)$ are
(A) $x + y = 4 \; and \;\; x - y = \frac{z}{8}$ (B) $x + y = 8 \; and \;\; x - y = \frac{z}{4}$
(C) $x - y = 4 \; and \;\; x + y = \frac{z}{8}$ (D) $x - y = 8 \; and \;\; x + y = \frac{z}{4}$

43. The equation $ax^2 + by^2 = 2cz, \;\; c \neq 0$ represents
(A) an ellipsoid (B) an elliptic paraboloid
(C) an elliptic cylinder (D) a hyperbolic cylinder

44. If α, β, γ be the roots of the equation $x^3 + px + q = 0, q \neq 0$ the value of $\sum \frac{1}{\beta+\gamma}$ is
(A) p (B) q (C) pq (D) $\frac{p}{q}$

45. The number of elements of the cyclic group of order 6 can be used as generators of the group are
(A) 3 (B) 1 (C) 2 (D) 4

46. **Statement A:** Every homomorphic image of a cyclic group is cyclic.
Statement B: Every isomorphic image of a cyclic group is cyclic.
(A) A is true only (B) B is true only
(C) Both A and B are true (D) Both A and B are false

47. If the radial and transverse velocities of a particle are always proportional to each other, then the equation to the path represents the nature
(A) Circle (B) Spiral (C) Ellipse (D) Parabola

48. A cubic tank is completely filled with water. What will be the ratio of the hydrostatic force exerted on the base and on any one of the vertical side?
(A) 1:1 (B) 2:1 (C) 1:2 (D) 3:2

49. The nine digits $1, 2, \ldots, 9$ are arranged in random order to form a nine digit number. The probability that 1, 2 and 3 appears as neighbors in the order mentioned is
(A) $\frac{1}{27}$ (B) $\frac{1}{36}$ (C) $\frac{1}{49}$ (D) $\frac{1}{72}$

50. A sales man has 80% chance of making a sale to each customer. The behavior of successive customers is assumed to be independent. If two customers X and Y enter the shop, the probability that the sales man will make a sale is
(A) $\frac{4}{5}$ (B) $\frac{16}{25}$ (C) $\frac{24}{25}$ (D) $\frac{3}{10}$